Autonomous On-Demand Transit Services over Existing Guideways

A thesis submitted toward the degree of
Master of Science in Industrial Engineering

by

Omer Karny

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Abstract

Technological advances in vehicle autonomy, vehicle connectivity and vehicle electrification are expected to revolutionize urban mobility by enabling seamless on-demand mobility services. Such advances will facilitate vehicle and journey sharing, allow for better traffic control, and increase the accessibility of citizens to existing mass transit systems. In this work, we present the potential of transforming guideway based public transit (tram, light-rail, regional trains, BRT) to novel on-demand point-to-point services. To achieve this, we propose coupling technologically advanced small autonomous vehicles with existing expensive underutilized network infrastructures. We identify and characterize types of existing public transit services that may benefit from such a transformation. We develop decision aid tools to support tactical and operational planning. Specifically, such tools will assist in defining the characteristics of the vehicle fleet to be used and will allow determining the passenger demand load the proposed services may endure at peak hours.

To examine the impact of the proposed transformation, we analyze the operations of the on-demand services over existing guideways and compare their performance to the currently offered public transit services. For this purpose, we develop two types of models. First, we develop an approximated representation of the dynamics of the system that is based on a batch-service queuing model. Such a representation allows fast performance evaluations and facilitate multiple comparisons and analyses. Second, we devise a finer representation of the system via a detailed event-based simulation model. The simulation model permits a comprehensive examination of various aspects of the system operations and allow a more thorough analysis of specific system settings. The results of this research prove that it is possible to satisfy demand while shorten waiting time of passengers by approximately 50% in some public transport systems. Furthermore, we succeed to measure more accurately the waiting times of passengers under a ridesharing policy, as compared to other mathematical models proposed in literature.
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List of Notations

$S$ Set of Stations
$t_{i,j}$ Travel time between station $i$ & $j$
$\lambda_{ij}$ rate of passengers who wish to travel from station $i$ to station $j$
$V$ Number of ACE-PRT vehicles
$C$ ACE-PRT capacity
$x_{ij}$ demand-driven rates of vehicles that travel between stations $i$ and $j$
providing service
$y_{ij}$ demand-driven rates of vehicles that travel between stations $i$ and $j$
relocating
$S^-$ All the stations with equal or bigger supply from station $i$ to station $j$
than the demand from station $j$ to station $i$ \{\{i | \sum_{j \in S} x_{ij} - \sum_{j \in S} x_{ji} \geq 0\}\}
$S^+$ All the stations with smaller supply from station $i$ to station $j$
than the demand from station $j$ to station $i$ \{\{i | \sum_{j \in S} x_{ji} - \sum_{j \in S} x_{ij} > 0\}\}
$D_i$ Rate of vehicles that should be relocated to station $i$
$O_i$ Rate of vehicles that should be relocated from station $i$
$P_{ij}$ The ideal proportion of vehicles that should be travelling between
stations $i$ and $j$
$\pi_{ij}$ The actual rate of vehicles that will be travelling between stations $i$ and

$\tilde{\lambda}$ Arrival of users – Israeli queue
$B$ Batch service time – Israeli queue
$E(w_{first})$ Sojourn time of the first user in the Israeli queue of each class
$E(s_{arb})$ Sojourn time of an arbitrary user
$\theta_{ij}$ Number of intermediate stops while traveling between station $i$ and
station $j$
$f$ Frequency of service
$\nu_f$ Greenshield’s Free speed
$k_j$ Greenshield’s Jam density
$k$ Greenshield’s density
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1. Introduction

Technological advances in vehicle autonomy, vehicle connectivity and vehicle electrification are expected to revolutionize urban mobility by enabling seamless on-demand mobility services. Such advances will facilitate vehicle and journey sharing, allow for better traffic control, and increase the accessibility of citizens to existing mass transit systems. Overall, such services promise to significantly improve the quality of service by providing “taxi-like” services at affordable costs (Chong et al., 2013; Spieser and Treleaven, 2014; Buehler, 2018). Nevertheless, technological, regulatory, moral, and cyber-security barriers are projected to be fully resolved only in two to three decades (Bansal and Kockelman, 2017; Fleetwood, 2017; Litman, 2018). Public Transportation (PT) today comprised mostly from a fixed-timetable, fixed route service, that has both pros and cons. The main strength of the public transport service is its purpose itself – offer the masses an affordable way to commute and travel internally and externally from the city. Some public transport type, such as metro and Bus Rapid Transit (BRT), offer not only an affordable way to commute but also an efficient way to commute due to good level of service which expresses in high frequency services. On the other hand, the weaknesses of PT are vast. PT services are cheap due to subsidized policy of governments (i.e., expensive) and as will shown later in this paper, it follows with high capital and operational costs (and even more expensive in rapid transit such as metro service). Added to the high cost of PT, some PT services offer low frequency service which lead to bad level of service. Fixed-route, fixed-timetable PT services exist in our life for a long time and had been explored widely and deeply. The service is optimized as it can be, and one might ask how much more it can be optimized in the current configuration.

New mobility services based on autonomous vehicle technology may be used at a large scale in dense city centers prior to the complete adoption of fully autonomous vehicles by deploying them in controlled environments. These are environments where interactions with pedestrians, human-driven vehicles, and other obstacles are limited and reduce the barriers of today’s adoption of autonomous vehicles. For example, in Personal Rapid Transit systems (PRT), small autonomous vehicles travel on guided pathways segregated from pedestrians and other traffic and provide passengers with on-demand point-to-point transportation between the stations of the system. The small size of the vehicles and automation enable them to operate at a higher average speed with considerably shorter headways compared to heavy rail, light rail, and bus systems. As a
result, the passengers’ total trip times can be reduced significantly. In addition, the short headways allow operating at peak-hours in capacities equivalent to light-rail and busways (Carnegie and Hoffman, 2007). At off-peak hours, PRT can scale down by reducing the number of operated vehicles without compromising service quality. Furthermore, previous studies have shown that providing mass transit services using a large fleet of small vehicles may result with lower operational and maintenance costs as compared to traditional public transit services (Anderson, 2000; Kerr et al., 2005; Tirachini et al., 2010; Juster and Schonfeld, 2013; Litman, 2015).

Despite these advantages, only five such systems currently operate worldwide (Staniscia, 2018). Low adoption can be attributed to the high investment required to establish an infrastructure for PRT, estimated at $20M to $50M per mile (Carnegie and Hoffman, 2007), and the challenge of integrating such systems into the existing urban landscape (Vuchic, 1996; Jaffe, 2014).

In this work, we examine the potential of transforming guideway based public transit (tram, light-rail, regional trains, BRT) to novel on-demand point-to-point services. To achieve this, we propose coupling technologically advanced small autonomous vehicles with existing expensive underutilized network infrastructures. Particularly, in the envisioned service, a fleet of small autonomous, electric vehicles will be installed on the existing infrastructure and will replace the existing fleet. A Passenger arriving at one of the systems’ stations, will inform the system about her/his desired destination and the system, in turn, will assign her/him to a vehicle which will provide direct service from the origin to the destination station. Contrarily to PRT, in this service, several passengers with the same destination that are waiting to be served, may be assigned to the same vehicle, subject to the vehicle capacity. In other words, the proposed service enables ridesharing that encourages this type of collective behavior.

The proposed transformation comes with several economic benefits. In particular, reduced operational costs due to the use of autonomous vehicles and reduced development costs due to the use of existing infrastructure. To further expose the advantages of the proposed system, in this study, we focus on quality-of-service measures. Specifically, we will measure the impact of various system characteristics and operation policies on the total journey time of the passengers. Our goal is to reveal settings under which the transformed systems will provide significantly higher level of service, as compared to the currently operating systems.
The station-based structure of many public transit services generates a positive demand concentration effect, facilitating a higher potential for a collective behavior, i.e., ridesharing. As the envisioned service will be provided at stations (existing and new), we wish to represent the supply/demand dynamics of the proposed on-demand service with appropriate queuing models. Previous studies on PRT systems, have represented such dynamics via standard queuing models, assuming a single user first-come-first-served regime (Lees-Miller, 2016). However, such approach ignores the potential of ridesharing. In this study, we propose better approximation of the waiting times under ridesharing policies by employing a first-come-first-served batch service queue, the so-called “Israeli Queue” (Boxma et al. 2008).

The contribution of this thesis is fourfold. First, we introduce a novel transformation concept that has not been studied before, i.e., utilizing existing public transit infrastructure to provide on-demand services via autonomous vehicles. Second, we develop an approximated representation of the dynamics of the system that is based on a batch-service queuing model. Such representation allows fast performance evaluations and facilitates multiple comparisons and analyses. Third, we devise a finer representation of the system via a detailed event-based simulation model. The simulation model permits a comprehensive examination of various aspects of the system operations and enables a more thorough analysis of specific system settings. Fourth, we analyze multiple case studies, compare the existing services to the proposed on-demand service, and characterize types of existing public transit services that may benefit from the proposed transformation.

The thesis is organized as follows: In Section 2, we review the current literature on public transit and on-demand services and particularly the PRT literature. Then, we review the current state of autonomous transportation services. Consequently, we provide a review of the quality-of-service measures common in the transportation literature and lastly, we identify the main gaps in the literature that this work is aiming to bridge. In Section 3, we present a queuing theory based approximate model which represents accurately the proposed on-demand service. In Section 4, we present the event-based simulation framework developed to represent in a finer resolution the dynamics of the existing public transit services and the proposed on-demand service. In Section 5, we present several real-world case studies and benchmark the existing services against the proposed on-demand services using the approximate model and the simulation model. Through these comparisons, we identify system characteristics under which the latter is
superior and highlight system configurations under which the approximate model is rather accurate. In Section 6 we conclude our findings and discuss several future research directions.

2. Literature review

Nowadays, the urban environment offers travelers a wide variety of shared mobility modes, including public transit, vehicle sharing services and on-demand transportation. This study proposes transforming some public transit services to on-demand services using Autonomous, Connected, Electrical PRT (ACE-PRT) vehicles. Accordingly, in this section, we review the transportation literature and position the proposed service with respect to the existing transit services. In Section 2.1, we review the public transit service literature and highlight the strengths and weaknesses of this transportation mode. In Section 2.2, we review the on-demand transit literature. In Section 2.3 we review the PRT literature. In Section 2.4, we review the state of the art in transportation services based on autonomous vehicles and the discuss the projections for the future, and particularly explain why this service will not be available in the near future. In section 2.5, we summarize the main quality of service measures commonly used in the transportation literature to better analyze the proposed ACE-PRT model. Finally, in Section 2.6 we outline the main gaps in the literature that this study is aiming to bridge.

2.1. Public transit

In this work, we refer to public transit as any transportation system that is available for use by the general public, providing transit services along fixed routes and operating according to predetermined schedules. In particular, this public transit type includes the following services: metro, train, tram, light rail and bus services.

Public transit services were first introduced to the world thousands of years ago, starting from ancient Egypt, taking a big leap with the steam engine into today’s electrified vehicles (Wootton et al. 1995; Train History – History of Rail Transport, 2020). The last decade has seen a decline in the usage of public transit. During 2014, public transit usage in Britain, France and Germany, was estimated to account for 12.8%, 14.9%, and 15.2% of the total trips performed, respectively. Some studies claim that public transit has reached a saturation point because it has not improved significantly for many years (White, 2016).
The management and operation of public transportation consists of strategic, tactical, operational and real time control planning phases (Desaulniers and Hickman, 2007). The strategic planning phase, a well-known and old problem, focuses on the design of the routes and networks so as to satisfy passengers’ demand (Lampkin and Saalmans, 1967; Silman et al., 1974; Hasselström, 1981; Magnanti and Wong, 1984; Cordeau et al. 1998; Schöbel, 2012). The tactical planning phase determines the frequency of the routes and the timetable for a given network design (Caprara et al. 2007; Yang et al. 2008; Fischetti et al. 2009; Niu and Zhou, 2013; Barrena et al. 2014; Hassannayebi et al. 2016). The operational planning phase typically concerns the construction of vehicle and crew schedules. Lastly, real time control relates to methods applied to recover from deviations/disruptions from the planned schedules due endogenous and exogenous factors (Cordeau et al. 1998; Caprara et al. 2007).

As mentioned before, PT was built to provide the masses a way to commute cheaply. This is the most significant strength of PT. Unfortunately, today’s PT solution has a lot of weaknesses as population grows and it comprises of 2 main things: low level-of-service and poor cost effectiveness. First, the level of service may be poor due to low frequency services, inappropriate routes for the demand and a high-demand-low capacity service during peak hours. The second reason is cost effectiveness (monetization and energy consumption). PT in the urban areas are financially not cost effective as it is either overused or underused (according to peak hours) and therefore inefficient and almost every PT service is losing money because of their inefficiency (Rodrique, 2016). As long as the urban population grows, today’s PT is becoming less utilize due to longer journey time, inefficiency (vacant spaces, Litman, 2015), high operating cost (Clark et al. 2007), and less relevant to more people due to fix guideways and timetables (Rodrique, 2016).

There are few other directions proposed for improvements such as minimizing the size measurements of urban transit vehicles as it is being recognized that infrastructure’s cost of small rapid transit is cheaper, energy efficient almost 3 times more than big rapid transit (Anderson, 2000; Kerr et al. 2005; White, 2016), and also cheaper 3 times more in operational cost (Anderson, 2000). This change in the vehicles’ measurements makes the smaller rapid transit potentially profitable (Anderson 1988; Anderson, 2000; Kerr et al. 2005; Tirachini, 2010; Juster and Schonfeld, 2013; Lees-Miller, 2016; Muller and Anderson, 2018).

The rural transit (e.g. heavy rail trains) had been critiqued for bad Level Of Service, LOS, (Velaga et al. 2012; Petersen, 2016) and lately is being criticized that it ought to
change its services in order to measure up with today’s transport alternatives (Rosenbloom, 2003). The transportation in rural areas are mainly consist of individual cars, low-frequency trains/buses and on-demand services such as dial-a-ride, shuttle vans, shared taxis (Velaga et al. 2012).

When comparing LOS and efficiency, urban PT services are more developed and more suited for the urban areas. The main reason for that is the variety of Rapid Transit (RT) systems available: Metros, light rail, Bus Rapid Transit (BRT), and Personal Rapid Transit (PRT). The rapid transit system today are offering high frequency mass transportation services e.g., Metros, BRT and in the future a new type of mass rapid transit such as Hyperloop (Taylor et al. 2016). With the rapid transit in hand, some optimizations are proposed such as flexible timetable in order to serve fully the passengers demands in the least time (Cadarso and Marin, 2012). Albeit, most of the urban PT is still comprised of buses, light rails, trams and has a lot of challenges and difficulties in the urban area. The most advanced PT, i.e., best level-of-service PT are the RT with high frequency service. The BRT is the top-tier (small to medium size) rapid transit PT which is available today in the world and proven to be relatively financially beneficial and with better journey time.

The BRT, according to the Federal Transit Administration (FTA), is “a high-quality bus-based transit system that delivers fast and efficient service that may include dedicated lanes, busways, traffic signal priority, off-board fare collection, elevated platforms and enhanced stations”. The BRT is sometimes referred in terms of level of service as a more flexible, less costly (capital and O&M) to the Metro transit when the guideway is exclusive, to the Light Rail Train (LRT) when the guideway is partially exclusive, and to a tram when there is no exclusiveness of guideway (Levinson et al. 2003). The BRT is using an existing Intelligent Transportation System (ITS) and modifying an existing infrastructure or rather build a new one but still is being operated as a fixed-route, fixed-timetable, non-autonomous mass transportation (Wright, 200). Furthermore, improvements for the BRT are mainly focused on building new guideways, purchasing more vehicles and upgrading traffic light synchronization (Currie, 2006; Hidalgo and Graftieaux, 2008; Nesmachnow et al. 2019). It seems BRT to be a good alternative for today’s cars and motorcycles as it is more comfortable than today’s cars / motorcycles when reducing journey time or when proving that it is more economically beneficial to use this type of transport (Satiennam, 2016). To conclude – the BRT has a lot of strengths but potentially a lot of weaknesses (Nikitas and Karlsson, 2015) mainly because BRT is
most likely to take a piece from the auto-mobile’s infrastructure which is in short, still some people might not see as a better way to transport and will not transfer from their own private car to use the BRT. All BRT related literature agrees that BRT is a flexible mass transit system which holds the advantages of a bus and the advantages of a rail transit. The literature also acknowledge that BRT is more cost-effective than heavy rail trains because of infrastructure and difference in size.

To conclude, building a new infrastructure for public transit with a fixed timetable, can be very complicated and not necessarily cost efficient or achieve best level of service. PT has a lot of weaknesses rather it is due to level of service or cost-efficient reasons. The top tier PT today is the BRT that satisfy both level-of-service constraint and cost effectiveness. Having said that, BRT services are just added to the current PT services and not replacing them. Therefore, one may observe that all other types of PT need to be re-examined for their efficiency and productivity and maybe be considered for replacement.

2.2. On-demand transit

In on-demand transit services, a fleet of vehicles is deployed to serve travel requests that are typically characterized by a pick-up location, a drop-off location and desired pick-up or drop-off time, often given in the form of a time-window. Multiple variates of the on-demand services have been applied in recent years. These systems can be categorized by the following attributes. First, vehicles may be restricted to travel on some predefined routes or may be allowed to take any path on the transportation network. Second, pick-up and drop-off locations may be limited to a finite set of locations in the service area (i.e. stations) or may be located at any point on the transportation network. Third, the arrival times of the vehicles to certain locations may be prescribed by predetermined schedules. Fourth, passenger requests may be known in advance or may only be revealed on-line while the system is operating. In other words, the underlying planning problem may be static or dynamic. Further analysis and characterization of various on-demand services can be found in Brake et al. (2007) and Errico et al. (2013).

On-demand transit is also known as Demand Responsive Transit (DRT). The definition of DRT is being used differently in the DRT literature (Dial-a-a-Ride, autonomous Dial-a-Ride, flexible transit service, mobility on demand, autonomous on-demand, flexible MoD etc.). DRT is referred to the utmost flexible transit system that mainly consider their passengers’ needs such as taxis (Luis Ferreira et al. 2007; Mulley
et al. 2009). Although the history of on-demand service starts with taxis as DRT, as technology developed, more DRT that serves masses amount of people succeeded to enable more flexibility in their use (Enoch et al. 2004).

Flexibility in on-demand transit may variate, e.g., when taxis are acting as ride-sharing services or buses that may take several passengers in a fixed route but will only stop in a station where there is a demand to pickup/drop-off. The Flexibility of a Transit System (FTS) is determined according to 5 main criteria: route, vehicle allocation, vehicle operator, type of payment and passenger category (Jenny Brake et al. 2006; Davison et al. 2012):

- Route – fixed route / free route which determined according to the passenger need.
- Vehicle allocation – frequencies of arrivals / allocating vehicle close to demand time.
- Vehicle operator – varied amount of operators / one operator type.
- Passengers category – special transport services / no restrictions.
- Type of payment – one way to pay fares / different ways to pay fares.

PT and DRT are both subsets of FTS where the PT is in the lower bound of the flexibility scale and DRT is in the upper bound. More DRT systems are now starting to show (e.g., yellow-taxi bus in the UK and minibuses shuttles in Tel-Aviv) and are technologically advanced as for cloud computation which is now available and allowing fast and on-demand computation (Mishra et al. 2012).

One of the most known, advanced, and flexible type of DRT is the Dial-A-Ride Problem (DARP). The DARP is comprised of assigning vehicle routes and time schedules, in a minimum cost, according to a given demand matrix, allowing ridesharing. The routes and timetable are not fixed as it changes according to the demand matrix, allowing great flexibility to the passengers. The standard DARP assumes both static and deterministic information. In real-life scenarios, those assumptions do not apply as user behavior is unexpected and information is stochastic (Molenbruch et al, 2017). A DARP with a fixed route, where only requests and trip journeys (due to congestion) is stochastic, are addressed mainly as a PRT services.

2.3. Personal Rapid Transit:
In PRT systems, small autonomous vehicles travel on guided pathways to provide passengers with on-demand transportation between the stations of the system, typically, direct origin-to-destination journeys with no intermediate stops. The infrastructure
consists of a main “highway” and “offline stations” (Carnegie and Hoffman, 2007). This layout and type of service enables the vehicles to travel with no interruptions (Juster and Schonfeld, 2013). Furthermore, the small size of the vehicles enables them to operate at a higher average speed with considerably shorter headways compared to heavy rail, light rail and bus systems. As a result, the passengers' total trip times can be reduced significantly. In addition, the short headways allow operating at peak-hours in capacities equivalent to light-rail and busways (Carnegie and Hoffman, 2007). At off-peak hours, PRT can scale down by reducing the number of operated vehicles, without compromising the quality of service. A review of the history of the system and its operating policies is provided by Raney and Young (2005).

In a survey on PRT deployment, Anderson (2000) concludes that PRT can function at a profit in small or large deployments and provide safe, reliable, all-weather transportation. Carnegie and Hoffman (2007) examine the potential viability of implementing PRT in New-Jersey. According to their report, PRT may provide considerably shorter journey times, may require relatively lower capital investment, and will be operated at reduced operational and maintenance costs due to the use of small automated vehicles. Juster and Schonfeld (2013), compared several transportation modes for the purple line in Washington, DC, including BRT, LRT and PRT. They conclude the PRT can provide better quality of service as compared to the alternatives at lower operational and maintenance costs.

Despite the advantages of PRT systems and its presentation more than 50 years ago, only five such systems currently operate worldwide: Morgantown (West Virginia, USA), Rotterdam (The Netherlands), Masdar City (Abu Dhabi), Heathrow Airport (UK), and Suncheon Bay (South Korea) (Staniscia, 2018). Low adoption of PRT can be attributed to the high investments required for its establishment and implementation (estimated in the range between 20 to 50 $M/mile (Carnegie and Hoffman, 2007)) and the challenge of integrating such systems into the existing urban landscape (Vuchic, 1996; Jaffe, 2014).

PRT has attracted scientific attention during the 1960’s and early 1970’s, where most studies focused on technological aspects in the design of the vehicles and the network, see Kovatch and Zames (1971), Cottrell (2005) and references therein. A ‘survey of PRT Vehicle Management Algorithms’ was presented in Priver (1974). However, as noted by the author, only a few of the 240 documents reviewed in that survey explicitly described algorithms applicable to the management of PRT systems.
New approaches regarding PRT optimization is elaborated in many research articles. One model proposes to model the design of a new PRT system as a combination of the Steiner problem and an assignment problem (Zheng and Peeta, 2015). The objective in this model is to minimize the guideway construction and user’s travel costs. The proposed model provides a solution in fast time (a network with 28 and 43 nodes is obtained by a Lagrangian-relaxation based algorithm in 30 minutes). Operational planning problem is also thoroughly discussed (Andréasson, 2003; Lees-Miller and Wilson, 2012). The objective of those research articles are to optimize the empty vehicle reallocation problem (measured by time / distance) and provide solutions such as heuristic methods. In another research, a stacker crane problem designed to manage a list of PRT requests solved with two Mixed Integer Linear Problem (MILP) formulations (Mrad and Hindri, 2015). The solution is optimized by minimizing the consumed energy. Few more research articles solve this problem with different approaches such as math-based constructive heuristic (Mrad et al., 2014), a honey-bee optimization algorithm (Fatnassi et al., 2016), and a MTZ and Flow-based solutions (Chebbi and Chaouachi, 2015). In all of these operational planning studies, ridesharing, congestion constraints and stochastic demand are ignored.

Ride sharing could be critical for PRT system capacity and passenger experience (Lees-Miller et al., 2009). For a simple point-to-point system, ridesharing can significantly reduce the fleet size while providing a high level of service to the passengers. In a more recent research (Lees-Miller, 2016), three lower bounds are described for the achievable mean passenger waiting time, based on an M/G/S queueing model, a heuristic for a static formulation and a Markov Decision Process (MDP) model. Also, in this research it is noted that the effect of ridesharing policies on passenger waiting times is not considered.

2.4. Autonomous vehicle services

Major advances in autonomous vehicle technologies have been made in recent years (Fagnant & Kockelman, 2014; Narayanan et al., 2020). Many major car corporation (such as Tesla, Volvo, Fiat, Toyota), technology companies and service providers (Moovit, Via, Google, Uber, Lyft) invest more and more funding and resources in the development of driverless vehicles. Fully autonomous vehicles are expected to improve urban transportation as we know it. By enabling higher vehicle utilization, better synchronization and many ridesharing opportunities, a high adoption of driverless vehicles may reduce by up to two thirds the total number of vehicles on the roads today.
In addition, this trend is expected to result with a significant reduction in traffic congestion and traffic fatalities (Currie, 2018).

Autonomous vehicles will enable a significant shift from privately owned vehicles to Mobility as a Service - MaaS (Narayanan et.al., 2020). Indications for this are given by the high participation of major mobility service providers in the development and testing of autonomous shuttles (UBER partners with Volvo, Google partners with Fiat, Intel acquiring Moovit). This has attracted the attention of the research community, multiple studies in recent years examine the planning and operation of Autonomous Mobility On Demand (AMOD) services (Narayanan et.al., 2020).

Having a promising footprint in the future, Shared Autonomous Vehicles (SAV) is been broadly explored in a variety of fields (Narayanan et.al., 2020): demand, fleet, traffic assignment, vehicle assignment, vehicle redistribution, pricing, charging and parking.

To determine demand in the SAV systems, it is commonly used to implement available data from different sources (Narayanan et.al., 2020). When data is not available, a Poisson distribution is implemented.

Vehicle assignment problem can be approached with a heuristic solution or an optimization algorithm (Narayanan et.al., 2020). The former is mostly implemented with a rule of assigning the nearest vehicle to the request. The latter is rarely used because of the complexity of the problem which may take too much compute time.

Vehicle repositioning, i.e., moving empty vehicles from low demand areas to high demand areas, is critical for optimizing these types of systems (Narayanan et.al., 2020; Vosooghi et.al., 2019). Four different approaches are proposed for solving the empty repositioning problem (Fagnant & Kockelman, 2014). The approaches are mainly moving unoccupied vehicles to adjacent blocks considering the size of the blocks, demand imbalance and randomness.

Nevertheless, the promising future of autonomous vehicles is still far away. Major obstacles for the full adoption of autonomous vehicles pertain to regulatory, security, privacy, and moral aspects (Fagnant and Kockelman, 2015; Bagloee et al., 2016; Bonnefon et al., 2016). A recent survey by Iclodean et al. (2020) lists the existing autonomous shuttle-based transit service. Almost all systems mentioned, operate only one or two autonomous shuttles on fixed route of a few kilometers. That is, these systems serve for testing and demonstration and are far from providing large scale transit services.

Recent projections regarding the autonomous vehicle market estimate that fully autonomous vehicles, i.e. SAE level 5 (Litman, 2017), will only become commercial at
the end on this decade. Furthermore, autonomous transportation is only estimated to become a dominant transportation mode in the three to four decades. This further emphasizes that for intermediate solutions that will enable the large-scale application of autonomous technologies in the very near future.

2.5. **Quality of service in public transportation**

The use of small autonomous vehicles is likely to result with significantly lower operational and maintenance costs. In particular, it will reduce significantly the need for driving staff and the small size of the vehicles will simplify considerably maintenance and repair operations (Tirachini et al., 2010; Juster & Schonfeld, 2013; Lees-Miller, 2016). Furthermore, the proposed transformation of existing infrastructures will require relatively minor capital investments. As the cost benefits are evident, the purpose of this study is to analyze and demonstrate the potential of on-demand services over existing infrastructures from a quality of service perspective. In this section, we briefly describe the main quality of service measures that are common in the transportation literature and focus on the measures that will be considered in this work.

The most impactful factors to determine QOS are short waiting times, accurate and reliable timetable, price, trip duration, crowdedness and number of transfers (Moovit, 2019). Other factors are also mentioned in the Highway Capacity Manual (HCM, 2010), and in the Fellesson et al. 2012 survey made in 9 cities in Europe. Some of the factors are quantitative and can be measured “objectively” by the system, other measures are more qualitative and are subject to the individual preferences of the travelers. Naturally, most of the planning models existing in the literature are oriented towards quantitative measures (Molenbruch et al, 2017). For example: total passenger travel time (Yang, 2008), total passenger waiting time (Lees-miller, 2016), and total number of transfers (Ferreira, 2007).
To conclude, we can see that the results from the field correlate with the “soft” factors which were assumed in the literature. Furthermore, Figure 1 displays the major factors for better LOS are waiting time and trip time, cost, comfort and safety. As can be observed, the major factors that influence passenger’s willingness to use public transportation are waiting time, trip time, cost, comfort and safety.

Recall that in proposed on-demand service vehicles will provide direct trips to the passengers. Clearly, minimizing the total journey time of the passengers in this system is equivalent to minimizing their waiting times at their origin stations. Therefore, in the approximate model presented in Section 3, we focus our attention on the total waiting time. However, the journey time of passengers in the exiting fixed route services is highly impacted by multiple stops at intermediate stations. Furthermore, in both types of services transfers are not considered. Therefore, to facilitate a meaningful comparison between the proposed services, we focus on the total journey time of the passengers. In other words, in this work, we take the total time passengers spend in the system as the main quality of service measure (Daszczuk et al. 2014; Shen and Lopes, 2015; Lees-Miller, 2016).

The arrival process of passengers to transportation systems are typically dynamic and stochastic. Therefore, to represent this nature and measure accordingly the waiting time, previous studies on on-demand services have either opted to use stochastic models, mainly queuing models, or have developed simulation models (Daszczuk et al. 2014; Shen and Lopes, 2015). With respect to queuing models, previous studies (e.g.: Lees-
Miller, 2016) have applied standard queueing models such as M/M/1 or M/G/S. These models enable a simplified representation of the service and the waiting time. However, a fundamental assumption in these models is that each service request (of a single passenger or a group of passengers) is arriving and being served separately, typically, following a first-come-first-served regime. Namely, standard queuing models do not allow representing the joint service of several service requests. Alternatively, batch service queuing model do facilitate this required property. In particular, in this work we adopt the “Israeli Queue” model of Boxma et al. (2008) in order to approximate the waiting time in the proposed on-demand service.

2.6. Literature gap:
This thesis proposes a new paradigm that may enable the implementation of large-scale autonomous transit services in the near future. Several aspects in the proposed service have never been studied before. First, the potential impact of transforming existing fixed route public transit services to on-demand services is a novel approach. More generally, most Operations Research studies on transportation services focus on means to improve existing systems rather than examining the potential of replacing them by completely new transit solutions. Second, the transportation literature provides several modeling frameworks for representing and planning semi-flexible systems (Koffman, 2004; Potts et al., 2010; Errico et al. 2013), the notion of on-demand transit over fixed routes was not considered in these frameworks. The relatively recent work of Pimenta et al. (2017) is the first and only to examine on-demand ride-sharing services over fixed routes. Lastly, the modeling of ridesharing services using batch service queuing models has not been done before. Such models facilitate more accurate approximation of waiting times in these highly stochastic systems. In particular, this work is the first to implement the Israeli Queue model in the context of transportation services.

3. An approximate queuing model
In this section we present an approximate queuing model for an on-demand transit service over fixed-route infrastructures. In section 3.1., we present the notations and the main assumptions made in the model. In section 3.2., we describe an approximate approach to calculate the rate of vehicles that enter and exit each station. In Section 3.3, we apply the Israeli Queue model to approximate the average waiting time and average journey time.
in the system. In Section 3.4 we approximate the journey times in the existing fixed-route service and present a comparison to the proposed on demand service.

3.1. Notations and assumptions
Consider a fixed-route public transit service connecting a set of stations $S$, numbered in ascending order, i.e., $1, 2, ..., |S|$, as depicted in Figure 2. In the transformed single-line system, the small autonomous vehicle can “turn around” at any station. In other words, when a vehicle exits a station it may travel oppositely to the direction it entered the station.

Let $t_{i,j}$ denote the direct travel time between stations $i$ and $j$. In particular, we assume that the travel time is the sum of direct travel times between all pairs of consecutive stations on the path between station $i$ and station $j$. That is, without loss of generality, for $i < j$ we have $t_{ij} = t_{ji} = \sum_{k=i}^{j-1} t_{k,k+1}$. In addition, we denote by $\lambda_{ij}$ the estimated rate of passengers who wish to travel from station $i$ to station $j$. Lastly, we denote by $V$ the number of small autonomous vehicles that will be deployed over the existing infrastructure and by $C$ their capacity.

![Figure 2- Illustration of a fixed-route public transit service with S stations](image)

We use the following simplifying assumptions in order to approximate the dynamics of the proposed service:

**Assumption 1**: the time required for the autonomous vehicles to enter a station, debark passengers, embark passengers and exit the station is negligible with respect to the travel times between the stations. In other words, these processes are assumed to be instantaneous.

**Assumption 2**: vehicles are assumed to be constantly travelling between stations of the system. In particular, a vehicle may be moving with passengers on board, namely conducting a service trip, or may be moving empty, for vehicle rebalancing purposes.

**Assumption 3**: As soon as a vehicle enters a station, it is assigned with the following destination station.

When passengers arrive at the station they announce their destination and enter a queue of passengers who wish to travel to that destination. When a vehicle departs to a certain destination, all waiting passengers who wish to travel to that destination embark the vehicle.
**Assumption 4:** in the approximate model, the capacity of the vehicles is non-binding.

**Assumption 5:** in the approximate model, the travel time on each segment is constant. In particular, the travel speed is not affected by the number of vehicles that travel on a segment, i.e., the speed is not impacted by congestion.

**Assumption 6:** for each station, the destinations of passengers wishing to travel from that station are uniformly distributed over all other stations in the system.

### 3.2. Approximate vehicle arrival rates

In this section, we propose an approach to approximate the rate of vehicles that enter or exit a station at a given period. Specifically, this requires estimating the rate of vehicles that provide service from/to a given station and the rate of empty vehicles relocated from/to a station.

We begin by approximating the rates of vehicles induced by the demand, disregarding the available fleet. Specifically, let \( x_{ij} \) and \( y_{ij} \) denote the demand-driven rates of vehicles that travel between stations \( i \) and \( j \), providing service and relocating, respectively. Based on Assumption 1, Assumption 2, Assumption 3, the rate of vehicles that enter a station must equal the rate of vehicles that exits that station. That is:

\[
\sum_{j \in S} x_{ij} + \sum_{j \in S} y_{ji} = \sum_{j \in S} x_{ij} + \sum_{j \in S} y_{ij} \quad \forall i \in S
\]  

(1)

An underestimation of the rate of vehicles that would be required to serve passengers between stations \( i \) and \( j \) is simply obtained by dividing the passenger arrival rate by the vehicle capacity, that is:

\[
x_{ij} = \frac{\lambda_{ij}}{C} \quad \forall i, j \in S
\]  

(2)

We note that some stations may exhibit higher demand for departing vehicles, i.e. as origin stations, while other stations may exhibit higher demand for arriving vehicles, i.e., as destination stations. To simplify the following discussion, we denote by \( S^- \) and \( S^+ \) the sets of stations that require more departing and arriving vehicles, respectively. Formally, the sets are defined as follows: \( S^- = \{i \mid \sum_{j \in S} x_{ij} - \sum_{j \in S} x_{ji} \geq 0\} \), \( S^+ = \{i \mid \sum_{j \in S} x_{ji} - \sum_{j \in S} x_{ij} > 0\} \). Furthermore, the rate of vehicles that should be relocated to station \( i \in S^- \) is denoted by \( D_i = \sum_{j \in S} x_{ij} - \sum_{j \in S} x_{ji} \). Similarly, the rate of vehicles that should be relocated from station \( i \in S^+ \) is denoted by \( O_i = \sum_{j \in S} x_{ji} - \sum_{j \in S} x_{ij} \). Next, in order to approximate the rates of vehicles that would be relocated between the stations we solve the following transportation problem.
\[
\min \sum_{i \in S} \sum_{j \in S} t_{ij} y_{ij} \tag{3}
\]

s.t.
\[
\sum_{j \in S^+} y_{ji} = D_i \quad \forall j \in S^- \tag{4}
\]
\[
\sum_{j \in S^-} y_{ij} = O_i \quad \forall i \in S^+ \tag{5}
\]
\[
y_{ij} \geq 0 \quad \forall i, j \in S \tag{6}
\]

The objective function (3) sums the total rate of vehicle travel time that is spent in relocations. Fulfilling this objective function will allow to serve demand in the fastest way. Constraints (4) and Constraints (5) determine the rates of vehicles that should be relocated to and from a station, respectively. Lastly, Constraints (6) set the non-negativity of the relocation rates.

Model (3)-(6) is essentially a transportation problem and therefore can be solved using the transportation simplex method. However, due to the structure of the network, a simple and efficient greedy procedure can be applied to solve it, Table 1 summarizes this procedure. Let \( \text{next}(i, S) \) be a function that returns the item that follows item \( i \) in the set \( S \).

Table 1: A greedy algorithm for the empty vehicle relocation problem

<table>
<thead>
<tr>
<th>Initialize:</th>
<th>( y_{ij} = 0 \ \forall i, j \in S ), ( o = \min{S^+} ), ( d = \min{S^-} ), ( \alpha = 0 ), ( \beta = \sum_{j \in S^-} D_i )</th>
</tr>
</thead>
</table>
| While \( \alpha < \beta \) | \begin{align*}
\text{If } D_d & < O_o \\
 y_{od} & = D_d \\
 O_o & = O_o - y_{od} \\
 d & = \text{next}(d,S^-)
\end{align*} | \begin{align*}
\text{else} & \quad y_{od} = O_o \\
 D_d & = D_d - y_{od} \\
 o & = \text{next}(o,S^+)
\end{align*} |
| | \( \alpha = \alpha + y_{od} \) |
| Return: | \( y_{ij} \ \forall i, j \in S \) |
Note that the solution obtained by the greedy algorithm satisfies the well-known no-crossing rule (McCann, 1999). Namely, for any $i, i' \in S^+$ such that $i < i'$ and any $j, j' \in S^-$ such that $j < j'$, $y_{i'j} > 0$ implies that $y_{ij} = 0$. Furthermore, it can be shown that a solution having the no-crossing property is unique. In other words, any solution that deviates from the left most $i \in S^+$ relocates to the left most $j \in S^-$ greedy rule, is bound to consist of crossings. As proven in McCann (1999) and references thereafter, considering the transportation problem on the line with convex cost functions, and particularly with a linear cost function, a solution satisfying the no-crossing rule is an optimal solution for the problem.

Recall that the demand-driven vehicle rates that were presented above, were calculated only with consideration of the passenger demand. Next, we adapt the rates taking into account $V$, namely, the actual number of small autonomous vehicles that will be deployed in the system. Let, $(x_{ij} + y_{ij}) \cdot t_{ij}$ represent the ideal number of ACE-PRT (the proposed solution) vehicles that will be travelling between station $i$ and $j$. The total number of vehicles that should ideally be deployed in the system is then obtained by summing this expression over all pairs of stations:

$$
\sum_{i \in S} \sum_{j \in S} (x_{ij} + y_{ij}) \cdot t_{ij}
$$

Furthermore, let $P_{ij}$ denote the ideal proportion of vehicles that should be travelling between stations $i$ and $j$, that is:

$$
P_{ij} = \frac{(x_{ij} + y_{ij}) \cdot t_{ij}}{\sum_{i \in S} \sum_{j \in S} (x_{ij} + y_{ij}) \cdot t_{ij}}
$$

Finally, we denote by $\pi_{ij}$ the actual rate of vehicles that will be travelling between stations $i$ and $j$ and approximate it, taking into consideration the existing fleet size, as follows:

$$
\pi_{ij} = \frac{V \cdot P_{ij}}{t_{ij}}
$$

### 3.3. Journey time calculation

The journey time of passengers in fixed route service consists of three primary components: (1) waiting time at the origin station (2) direct travel times between the stations (3) stopping times, i.e. the times required to embark and disembark passengers at
intermediate stations. Recall that the ACE-PRT provides direct services, therefore, the journey time solely consists of (1) and (2). While (2) is predetermined (Assumption 5), the main challenge here is to estimate the waiting time of each passenger at his/her origin station, considering that ridesharing is allowed.

Under a ridesharing policy, several users with the same destination, who arrived to the origin station at different times, may be served by the same vehicle. We wish to take this into account while approximating the expected waiting time at each station. For this purpose, we utilize the Israeli Queue model to approximate the waiting time at a single station and repeat the calculation for all stations.

The Israeli Queue is a batch-service queue that follows a First-Come-First-Served regime. Users who arrive to the queue are assumed to belong to one of \( N \) different classes. When the server begins to serve the first user in the queue, it serves at the same time all other users of the same class waiting in the queue. Specifically, there is no limitation on the number of users that can be served simultaneously (as in Assumption 4). The service time is assumed to be independent of the number of users that are served simultaneously. In other words, the service rate is defined per batch, independent of its size. Furthermore, the Israeli Queue assumes **gating in the end**, i.e., users arriving at the queue while their class is being served are assumed join the served batch without any waiting time.

The model assumes for each class that the arrival of users to the queue follows a Poisson process with rate \( \tilde{\lambda} \) (identical for all classes). In addition, the batch service time, \( B \), is assumed to be exponentially distributed with mean \( 1/\tilde{\mu} \) (also identical for all classes).

The sojourn time in the Israeli Queue is calculated by first representing the sojourn time of the first user in the queue of each class. The waiting time of the first user is Erlang distributed, considering this, the expected waiting time of the first user is:

\[
E(w_{first}) = \frac{\tilde{N} - 1}{\tilde{\mu}} \left( 1 - \frac{K^{(\tilde{N}-1)}}{K^{(\tilde{N}-2)}} \right)
\]

where \( K^{(\tilde{N})} = \left( \sum_{k=0}^{\tilde{N}} \tilde{\rho}^{k\tilde{N}} / (\tilde{N}-k)! \right)^{-1} \) and \( \tilde{\rho} = \tilde{\lambda} / \tilde{\mu} \). Furthermore, the sojourn time of the first user is given by:

\[
E(s_{first}) = E(w_{first}) + \tilde{B}
\]

Next, the sojourn time of an arbitrary user in the queue is calculate by:
The waiting time of passengers in a single station of the ACE-PRT system is approximated using the Israeli Queue model as described in what follows. The arrival rate of passengers wishing to travel from station $i$ is denoted by $\lambda_i$. By Assumption 6, the destinations of passengers wishing to travel from station $i$ are uniformly distributed over all other stations in the system. Namely, for any other station $j$, the rate of passenger wishing to travel from station $i$ to station $j$ is $\lambda_{ij} = \frac{\lambda_i}{|S|-1}$.

We assume that at the same moment a vehicle departs from a station, a new vehicle is assigned to serve the next destination (class). The waiting time of the passengers who will board this vehicle consists of the time spent waiting in line until this vehicle was assigned (equivalent to waiting in the Israeli Queue) and the time until the assigned vehicle arrives at the station (equivalent to service time in the Israeli Queue). Using (8), the time until a new vehicle arrives in station $i$, is given by $B_i = \left(\sum_{j \in S \setminus \{i\}} \mu_{ji}\right)^{-1}$. Lastly, recall that the Israeli Queue assumes gating in the end. With respect to the ACE-PRT system, passengers who arrive at the station prior to the assigned vehicle and share the same destination, will board it. In Table 2, we summarize the relations between the notations of the Israeli Queue model and notations of a single station in the ACE-PRT model.

<table>
<thead>
<tr>
<th>Israeli Queue</th>
<th>Station $i$ in the ACE-PRT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{N}$</td>
<td>$</td>
</tr>
<tr>
<td>$\bar{\lambda}$</td>
<td>$\lambda_i$</td>
</tr>
<tr>
<td>$\bar{\mu}$</td>
<td>$\mu_i = \sum_{j \in S \setminus {i}} \pi_{ji}$</td>
</tr>
<tr>
<td>$\bar{B}$</td>
<td>$B_i = \left(\sum_{j \in S \setminus {i}} \pi_{ji}\right)^{-1}$</td>
</tr>
</tbody>
</table>
To conclude, let $E(s_{arb}^i)$ denote the expected waiting time of passengers that wish to depart from station $i$. The expected journey time of passengers departing from station $i$ is then given by:

$$E(s_{arb}^i) + \frac{\sum_{j \in S \setminus \{i\}} t_{ij}}{|S| - 1}$$

By summing (9) over all stations in the ACE-PRT system, and considering the rate relative demand for each station, the expected journey time in the system can be derived:

$$\sum_{i \in S} \lambda_i \left( E(s_{arb}^i) + \frac{\sum_{j \in S \setminus \{i\}} t_{ij}}{|S| - 1} \right)$$

Next, we briefly present an approximate model for the journey time in fixed route services, which is later used for comparison purposes. In fixed route services, it is common to approximate the waiting as half the time interval between consecutive vehicle arrivals (for example, see Barrena et al. 2014). Let $f$ denote the frequency of service, then the expected time between vehicle arrivals is $\frac{1}{f}$ and the expected waiting time is approximated by $\frac{1}{2f}$. Furthermore, in fixed route services, a non-negligible portion of the journey time is due to intermediate stops. Let $\Delta$ denote a constant time required for each stop, and let $\theta_{ij}$ denote the number of intermediate stops while traveling between station $i$ and station $j$. That is, the time spent due to intermediate stops between stations $i$ and $j$ is $\Delta \theta_{ij}$. Lastly, the direct travel times between stations $i$ and $j$ is denoted by $t_{ij}$, as in the ACE-PRT model. The expected journey times of passengers who wish to travel from station $i$ to station $j$ is then given by: $\frac{1}{2f} + \Delta \theta_{ij} + t_{ij}$. The expected travel time of an arbitrary passenger in the fixed route services is:

$$\sum_{i \in S} \sum_{j \in S \setminus \{i\}} \lambda_{ij} \left( \frac{1}{2f} + \Delta \theta_{ij} + t_{ij} \right)$$

3.4. Analytical comparison

In this section, we present an initial analytical comparison between the approximated ACE-PRT model and the fixed route service. For this purpose, we present additional assumptions that enable further simplifying the model and highlighting the main differences between the two types of services. In particular, we study a special case in
which the travel time between any pair of neighboring stations is identical, i.e. \( t_{i,i+1} = t \ \forall i \in \{1 \ldots |S| - 1\} \) and the rates of passenger wishing to travel between any pair of stations are identical, that is \( \lambda_{ij} = \lambda \ \forall i, j \in S: i \neq j \).

Under these assumptions, Equation (2) reduces to \( x_{ij} = \left\lceil \frac{\lambda}{t} \right\rceil \ \forall i, j \in S \). As a result, due to the fully symmetric demand pattern, there is no need for relocating empty vehicles, that is \( y_{ij} = 0 \ \forall i, j \in S \). Thus, Equations (7)-(8) reduce to:

\[
\begin{align*}
P_{ij} &= \frac{\left(\left\lceil \frac{\lambda}{t} \right\rceil\right) \cdot t \cdot (\theta_{ij} + 1)}{\sum_{i \in S} \sum_{j \in S \setminus \{i\}} \left(\left\lceil \frac{\lambda}{t} \right\rceil\right) \cdot t \cdot (\theta_{ij} + 1)} = \frac{(\theta_{ij} + 1)}{\sum_{i \in S} \sum_{j \in S \setminus \{i\}} (\theta_{ij} + 1)} \quad (12) \\
\pi_{ij} &= V \cdot \frac{1}{\sum_{i \in S} \sum_{j \in S \setminus \{i\}} (\theta_{ij} + 1)} \cdot \frac{1}{t} \cdot (\theta_{ij} + 1) = \frac{V}{t} \cdot \frac{1}{\sum_{i \in S} \sum_{j \in S \setminus \{i\}} (\theta_{ij} + 1)} \quad (13)
\end{align*}
\]

Hence, the rate of vehicles that enter station \( i \), in this special case is:

\[
\mu_i = \sum_{j \in S \setminus \{i\}} \pi_{ji} = \frac{V}{t} \cdot \frac{|S| - 1}{\sum_{i \in S} \sum_{j \in S \setminus \{i\}} (\theta_{ij} + 1)} \quad (14)
\]

Noting that \( \theta_{ij} = |j - i| - 1 \), the sum in the denominator of Equation (14) can be written as:

\[
\sum_{i \in S} \sum_{j \in S \setminus \{i\}} (\theta_{ij} + 1) = 2 \cdot \sum_{i=1}^{\lfloor \frac{|S|}{2} \rfloor} \sum_{j=i+1}^{\lfloor \frac{|S|}{2} \rfloor} (j - i) = 2 \cdot \sum_{i=1}^{\lfloor \frac{|S| - 1}{2} \rfloor} i \cdot (|S| - i) = \frac{(|S| + 1) \cdot |S| \cdot (|S| - 1)}{3}
\]

Reinserting this expression in to Equation (14), we obtain:

\[
\mu_i = \frac{V}{t} \cdot \frac{3}{(|S| + 1) \cdot |S|} \quad (15)
\]

Finally, the expected time between vehicle arrivals at station \( i \), in this special case, is:

\[
B_i = \frac{(t \cdot (|S| + 1) \cdot |S|)}{3V} \quad (16)
\]

Observing Equation (16), it becomes evident that in the case of equally distributed stations with uniform arrival rates of passengers (i.e. the special symmetric case), the expected time between arrivals:

- Increases linearly with the travel time between neighboring stations
- Decreases linearly with the number of vehicles distributed in the system
- Increases quadratically with the number of stations in the system.
Next, we compare the expected journey times in the ACE-PRT system and the fixed route services under the special symmetric case. For the ACE-PRT, Equation (10) reduces to:

\[
\sum_{i \in S} \lambda \left( |S| - 1 \right) \left( E(s_{arb}^i) + \frac{t \sum_{j \in S \setminus \{i\}} \left( \theta_{ij} + 1 \right)}{|S| - 1} \right) \frac{\sum_{i \in S} \lambda \left( |S| - 1 \right)}{\sum_{i \in S} \lambda \left( |S| - 1 \right)}
\]

\[
= E(s_{arb}) + \frac{\lambda \cdot t \cdot \left( |S| + 1 \right) \cdot |S| \cdot (|S| - 1)}{3 \lambda \cdot |S| \cdot (|S| - 1)} = E(s_{arb}) + t \cdot \frac{(|S| + 1)}{3}
\]

For the fixed route services, Equation (11) reduces to:

\[
\sum_{i \in S} \sum_{j \in S \setminus \{i\}} \lambda \left( \frac{1}{2f} + \Delta \theta_{ij} + t(\theta_{ij} + 1) \right) \frac{\sum_{i \in S} \sum_{j \in S \setminus \{i\}} \lambda}{\sum_{i \in S} \sum_{j \in S \setminus \{i\}} \lambda}
\]

\[
= \frac{\lambda \left( |S| (|S| - 1) \right) \frac{2f}{2f} + \Delta |S| \cdot (|S| - 1) \cdot (|S| - 2) + t \left( |S| + 1 \right) \cdot |S| \cdot (|S| - 1)}{\lambda |S| (|S| - 1)}
\]

\[
= \frac{1}{2f} + \Delta \frac{(|S| - 2)}{3} + t \frac{(|S| + 1)}{3}
\]

From the obtained equations it becomes evident that the expected onboard travel time is identical in both systems, and therefore it is sufficient to compare the remaining components of the journey time. Namely, the waiting in the ACE-PRT system should be benchmarked against the expected waiting time in the fixed route service plus the expected time spent during intermediate stops.

To further explore the differences analytically, the expected waiting time in the ACE-PRT system should be expressed in terms of the system parameters. However, the expressions obtained for small “toy” systems are already hard to handle. To demonstrate this, we present in Table 3, the expected waiting times as a function of \( B \) and \( \lambda \) for systems with two and three stations. A comparison for systems with more stations is performed numerically. The outcome of this numerical comparison is presented in Section 5.
Table 3: the approximated average waiting times for “toy” systems with 2 and three stations

| $|S|$ | $E(s_{arb})$ |
|-----|---------------|
| 2   | $\frac{4\lambda^2 B^3 + 5\lambda B^2 + 2B}{2\lambda^2 B^2 + 1 + 2\lambda B}$ |
| 3   | $\frac{20\lambda^2 B^3 + 9\lambda B^2 + 2B + 10\lambda^3 B^4 + 8\lambda^3 B^3}{12\lambda^2 B^2 + 2 + 6\lambda B + 12\lambda^3 B^2}$ |

To conclude, as the number of stations increase the expected waiting time in the ACE-PRT increases significantly (at least quadratically) while the expected time in fixed route service increases moderately with respect to number of stations and the intermediate stop time. Based on these insights, we conjecture that the transition to ACE-PRT systems will be beneficial in terms of the passenger journey times in cases where the travel time between consecutive stations is not very long, the stopping times are significant and, most importantly, the number of stations on the line are not too many. Naturally, as the number of ACE-PRT vehicles increases, better service can be provided.

In the following section, we present a simulation model developed to enable a more detailed analysis of the ACE-PRT system. In Section 5, using the simulation, we further explore scenarios in which the ACE-PRT system may be superior based on data obtained from real-world systems.
4. An Event-Based Simulation Model

In this section we present an event based simulation model that represents in finer details the operations of the ACE-PRT services. This includes: general passenger arrival processes, passenger assignment policies and empty vehicle relocation strategies. This simulation model is devised in order to validate the results obtained by the approximate model while relaxing several modeling assumptions made in the approximate model. In particular, it will enable testing the proposed services under multiple system settings. Namely, we case study several existing real-world systems and examine how the ACE-PRT services perform under changing demand loads while varying the fleet sizes, the vehicles capacities, and the required vehicle headways.

4.1. Input

The event-based simulation model is based on 5 main pillars: system infrastructure, vehicles, demand matrix, passenger behavior, and reallocation policy of empty ACE-PRT vehicles. In the following paragraphs, we will describe the 5 input pillars.

The system infrastructure input is represented as a line of stations. The line of stations is configured according to the number of stations, distance between each pair of stations, number of embark ing berths, number of waiting berths, and segment capacity (of vehicles) between each 2 adjacent stations. To avoid congestion, segment capacity between 2 adjacent stations will be determined by the Greenshields function (Van Aerde and Rakha, 1995; Rakha and Crowther, 2002): \( v_f = \frac{v_f}{k_j} k \), where: \( v_f \) is the free speed, \( k_j \) the jam density, and \( k \) is density. To use the Greenshields function, we determine the free-speed velocity and the jam-density. As to the definition of a station in the system, each station is well described by the following attributes: number of embark/alight berths, 1 waiting berth with unlimited capacity (in case all embark/alight berths are full).

The second pillar of the event-based simulation model is the vehicles. The vehicles are the “servers” in this type of model and the fleet size is equivalent to the number of “servers”. Each vehicle will be represented by: vehicle capacity (how many passengers per a ride is allowed), vehicle speed, current position, future position. The vehicle velocity will be determined according to the average speed of the existing public transportation vehicles which currently operate in the infrastructure case studies. The initial position of a vehicle is manually determined in the beginning of the simulation. Future position of a...
vehicle is determined by either passenger destination request or according to the empty vehicle relocation policy.

The third pillar is the passenger requests. In particular, the passenger requests are represented by Origin-Destination matrices, which represent the arrival rate of passengers wishing to travel between each pair of Origin-Destination stations, at each period of the day. Throughout this study, we assume the passenger arrivals follow a Poisson process.

The fourth pillar is the passenger behavior. Arriving passengers leave the system in one of two options. The first, they exit the system in their destination station after being served. The second, passengers who wait in their origin station more than a predetermined abort time, abandon the system without being served. In the simulation, we monitor the number of aborting passengers as one of the key performance indicators of the system.

The fifth and last pillar is the reallocation policy of the empty ACE-PRT vehicles. Every fixed period of time, the empty cabin relocation procedure will be according to the proposed algorithm in section 3.2. The reallocation will take place repetitively after an arbitrary amount of time. Because the system is dynamic, and supply & demand for each station changes rapidly, an estimated demand forecast to optimize the reallocation policy is considered.

4.2. Performance measures
One of the goals of the thesis is to explore if the new proposed system will increase the quality of service from the current systems that exist today. As seen in section 2.5, 60% of the survey participant declared that shorter waiting time, shorter journey time, less crowdedness, on-time schedule, good pricing and less transfers will increase the quality of service and therefore will increase the use of public transportation. The ACE-PRT system, by definition, decreases the number of transfers to 0. Accurate schedule and pricing, may also be optimized with a proper implementation of the ACE-PRT system. The measurements that needs to be verified is the journey time, crowdedness, and the waiting time of passengers (average and max waiting time) in certain use cases. To also determine the type of use-cases where the proposed system is feasible (i.e., with no traffic jams) and better-perform, the following use-cases parameters will be monitored: total number of passengers in the system, number of passengers leaving the system without being served, average vehicle utilization (vacant / occupied), number of passengers per a trip distribution, segment occupancy to ensure realistic vehicle flows and speeds that
allow safely operating the system with the Greenshields model (Van Aerde and Rakha, 1995; Rakha and Crowther, 2002), and the time the system is under-saturated and over-saturated. The last two measurements, is to decide if the system performance is feasible. If the time the segments are over-saturated with vehicles for a significant amount of time, the system is clogged and the assumed velocity of vehicles is no longer valid and therefore, the proposed model is not feasible for the specific use-case.

4.3. Main Framework:
The event-based simulation framework is comprised of an initialization phase, the main event-based procedure, and a termination process. The simulation is driven by an event list, a time ordered set of events that are planned to occur in the simulation. At each step the earliest event in the list is taken out and it triggers a process that may change the state of the system. Such process typically also leads to the creation of new events that are inserted to the event list. This iterative procedure continues until some stopping criteria are met. Then, a post-processing procedure is triggered and the main performance measures are returned. In the initialization phase, the various input components are incorporated, including: infrastructure characteristics, vehicle attributes (including their initial positions), Origin-Destination matrices representing the passenger requests. Using this information, an initial list of passenger arrival events is created. A flow chart representing the main simulation procedure is presented in Figure 3. The notations used to describe the framework and its components are summarized in Table 3.

![Figure 3 - Event-based simulation main framework](image)
Table 3: notations of the system’s states

<table>
<thead>
<tr>
<th>Notation</th>
<th>Attribute</th>
<th>State</th>
</tr>
</thead>
</table>
| $S_s$    | Station   | • Vacant vehicles in berths  
          |           | • Future arrivals of vehicles  
          |           | • Passenger queue  
          |           | • Vehicles in waiting berths  
          |           | • Arrival list of passengers  
          |           | • Total current demand |
| $F_f$    | Vehicle   | • List of passengers  
          |           | • Last vacant time  
          |           | • Last occupant time  
          |           | • Total vacant time  
          |           | • Velocity  
          |           | • Starting station  
          |           | • Current station  
          |           | • Future station  
          |           | • Current segment |
| $P_p$    | Passenger | • Enter time to station  
          |           | • Time until served  
          |           | • Exit time from station  
          |           | • Origin station  
          |           | • Destination station  
          |           | • Projected abort time  
          |           | • Served by vehicle |
| $l_{system}$ | Total number of passengers in the system thus far | $0 < l_{system} \leq \text{Total arrival list}$ |
| $l_{model}$ | Current number of passengers in the system | $0 \leq l_{model} \leq l_{system}$ |
| $T$      | Model run time | $0 \leq T \leq \text{Simulation time limit}$ |
| $T_{waiting time}$ | Total waiting time of passengers | $0 \leq T_{waiting time} \leq \sum_{i=1}^{[P]} \text{Abort time}$ |
| $M_{waiting time}$ | Max waiting time | $0 \leq M_{waiting time} \leq \text{Abort time}$ |
| $N_{aborts}$ | Number of aborts | $0 \leq N_{aborts} \leq |P|$ |
| $T_{vacant time}$ | Total vacant time of vehicles | $0 \leq T_{vacant time} \leq \sum_{i=1}^{[F]} T$ |
| $N_{trips}$ | Number of trips | $0 \leq N_{trips}$ |
| segment$_i$ | Current number of vehicles on segment $i$ | $0 \leq \text{segment}_i \leq |F|$ |

Six types of events drive the simulation: (1) passenger arrival, (2) vehicle entry to a station, (3) vehicle entry to a station’s berth, (4) reallocation of empty vehicles, (5) passenger abort and (6) road-segment update. Each of these event types triggers a different process, as will be described in the following section. The relation between these event types is summarized by an events graph if Figure 4.
4.4. Initialization & Processes:

In the initialization phase, attributes of the system, infrastructure, and vehicles are determined. For the system values, stations are constructed, each with an arrival list according to the demand matrix, number of embarking berths according to length of stations, and unlimited number of off-line waiting berths. As part of the system and infrastructure, distance between stations matrix is imported according to the case study. For each segment between two adjacent stations, the saturation limit is calculated according to the Greenshields model. The vehicles attributes that are set include a passenger capacity limit, average velocity and initial position. Then, an initial event list is created consisting of passenger arrival events, each passenger is characterized by an arrival time, origin station, destination station, and waiting time limit. The event list also contains a reallocation event, which generates the following relocation event, in a periodic manner. Vehicles are distributed uniformly across stations. Lastly, the performance measures and time indicators are initialized. The initialization process is summarized in Figure 5.
Next we describe the processes that are triggered due to the occurrence of each event type. The first event type represents a passenger arrival to the system. The arrival of a new passenger is determined according to the arrival list, constructed according to the demand matrix per use case. When a new passenger arrives to the system, the passenger enters its origin station with a declared destination station and abort time. In accordance, a passenger abort event (event type #5) is added to the event list. With the entry of the passenger, the total number of passenger in the system and the current number of passengers in the model is updated accordingly. Once the passenger enters the station, the passenger will continue in one of two possibilities: (1) waiting in the station to board a vehicle by opening a new queue of passengers or joining an existing queue with the same destination (2) being served immediately by a vacant vehicle. This creates two new events to the event list: A vehicle entry to a station (event type #2) , and Road-segment update (event type #6). In addition the following performance attributes are updated: total waiting time of passengers, total vacant time of vehicles, number of trips and current number of vehicles on a segment. Finally, when a passenger is served, the vehicle departs from the origin station and clears a berth, if there are vehicles waiting in the waiting berth of the origin station, a vehicle entry to a berth event (event Type #3) is added to the event list to be executed immediately. The process that is triggered due to the occurrence of a passenger arrival event is presented as a flow chart in Figure 6.
The second event type is vehicle entry to a station (event type #2). When a vehicle enters a station, if the station is not full with vehicles, the entry of a vehicle to berth will be added to the event list and will be executed immediately (event type #3). If the station is overloaded with vehicles, the vehicle will enter the waiting berth and wait to its turn to enter the station’s berth, see Figure 7.

The third event type is the vehicle entry to a berth. A vehicle that enters a berth might be with or without passengers on board. If passengers are on board, they embark immediately, their elapsed journey time is updated, and the vehicle becomes vacant. Following, if there are waiting passengers at the station, they will embark immediately.
the vehicle that has just become vacant. This leads to the creation of two new events: vehicle entry to a station (event type #2) and road-segment update (event type #6). The process that is trigged due to the occurrence of vehicle entry to a berth is presented in Figure 8.

The fourth event type represents the periodic reallocation of empty vehicles. The empty vehicle reallocation is operated according to the greedy algorithm presented in section 3.2. This process is executed recurrently according to a predefined time interval. That is, when such an event of this type occurs, it generates the following event of this time, a time interval later. The process begins with the calculation the desired redistribution of empty vehicles. According to the result of the desired distribution, empty vehicles are designated to their new stations. For each vehicle to be relocated, two new events are added to the event list: road-segment update and vehicle entry to a station. See Figure 9.
The fifth event type is the passenger abort. For every arriving passenger, an abort time threshold is defined, representing the maximum time the passenger is willing to wait for a vehicle before she abandons the system. When this event occurs, if the passenger has not been served yet, she is removed from the passenger queue at the origin station, and the number of aborting passengers is increased by one.

The sixth and last process is the Road-segment update. Updating continuously the segment-vehicle load is crucial to determine if the system is over-saturated with vehicles, and to ensure there is a free flow velocity of vehicles. Every segment has its limit for number of vehicles that it can hold, and once the number of vehicles on that segment is passing this limit, it is monitored by the simulation, including the duration of such oversaturation. At the occurrence of this event, i.e. the entrance to the segment, we check, whether this the last segment on the current route of the vehicle. In case not, a new
Road segment event is inserted to the event list, representing the entrance time to the following segment, see Figure 11.

**Figure 11 - Event-based simulation road-segment update**

5. Numerical Experiments

In this section we describe the numerical experiment we have conducted in order to test the proposed service, using both the approximate model and the simulation model. In Section 5.1, we present the systems we have used as case studies. We define the values of the global parameters used throughout the numerical experiment and detail system specific values for each case study. In Section 5.2 we present and discuss the obtained results.

5.1. Case studies

We have selected several systems to use as case studies, based on available data and so as to create a variety of systems. That is, our aim was to examine a range of settings, considering the number of stations on the line, the distance between the stations, the arrival rate of passengers and various demand patterns. In particular, we have tested the following systems: Haifa Metronit red line, Tel-Aviv suburban train, Berlin M2, Berlin M4, Boston Blue Line, Boston Orange Line, Bukarest M41.

The case studies are based on data from official sources, mainly focusing on high-demand rush hour periods to test the proposed model under challenging conditions. This provides a minimum benchmark for the model's service quality, as it does not rely on predicted demand or rush hour patterns. Particularly, the attributes of the proposed simulation model were determined in a conservative manner, to ensure no advantages will be given to the proposed service.
The number of stations and distance between stations are determined by the observed infrastructure case study in the simulation. Number of embarking berths per station were determined according to a rough estimation of \( \frac{\text{Length of the existing public transportation vehicle}}{\text{Length of the proposed ACE-PRT vehicle}} \). Specifically, “Metronit” – 18.75m, Tram / MetroTram / Subway – 40M, suburban train – 187m and ACE-PRT=3.74m. Furthermore, we assume that there is unlimited number of wait berths per station, in order to simplify other constraints and complexities which are not in the scope of work in this simulation.

The free speed will be assigned according to the max velocity of the TPT. Because jam-density is determined according to observations, we use former observations and determine \( k_j = 80 \frac{\text{vehicles}}{\text{KM}} \).

The vehicles capacity (10 / 20 passengers), velocity (~30KPH), and length (3.74 meters) were chosen according to existing similar vehicles, such as the existing PRT vehicles. To analyze the vehicle capacity impact on the results, we have also examined vehicle capacity of 99 passengers, in order to represent a strictly non-binding capacity scenario. In addition, empty vehicle repositioning is set to be triggered every 15 minutes.

The demand data is described for each system in the following subsections. In cases of heterogeneous demand, when applying the approximate model, the demand matrices are modified such that for every origin station the total arrival rate is uniformly distributed over all destination stations, as follows:

\[
\sum_{j \in S \setminus \{i\}} \lambda_{ij} / |S| - 1
\]

Lastly, the abort model is configured so that every passenger that is not being served by an ACE-PRT for more than 60 minutes since the passenger arrival, will exit the system. When exiting the system, the passenger’s waiting time will not be considered in the system’s average waiting time calculation, and the \( N_{\text{aborts}} \) parameter will be incremented by 1.

### 5.1.1. Metronit – Red Line

The first case study is a BRT service characterized by many stations and short distance between stations (Levinson et al. 2003). The “Metronit” is a BRT operating in Haifa, Israel, and serves 100,000 passengers riding the system per a day (Ynet, 2015) in five operating lines. For the most part, the BRT has a segregated and slightly elevated (7 CM)
path. The Metronit does not have the right of way when intersecting with regular roads. The Metroint drives according to a pre-determined line, and stops at each station, regardless if passengers would like to ascend/descend the bus.

In the following simulation, the proposed model will be compared to the Metronit Red line consisting of 38 stations spread over 25 KM with an approximately 1-minute drive between station. The average rate of vehicle arrivals during peak hours in the Red line is between 4-8 minutes, and with an average velocity of 23 KM/H. As there is no real-life information about the demand matrix for this system, we assumed a uniform distribution of origin-demand matrix, so the estimated average arrival rate of passengers for each station is approximately \( d_{i,j} = 5 \forall i \neq j \in S \).

5.1.2. Tel Aviv Suburban Train
The second case study is the Tel Aviv suburban train system characterized with large amount of demand, small amount of stations and long distances between station. The Israeli suburban train operating on an elevated railway allowing an undisturbed transit. The train’s intermediate coach are the Bombardier TWIINDEXX Double-Deck Trains which each contain 121 passengers’ seats. The Israeli train has 2 locomotives in each train: Euro4000 of Stadler company with no passenger seats, and the Bombardier TWIINDEXX Double-Deck driving coach – 70 seats.

One train contains between approximately six intermediate coaches and two locomotives so in total the Israeli train may contain 790 passengers’ seats, in the length of 187 meters.

We focus on the line segment from Rosh Haayin to Hagana station, Tel-Aviv. The line consists of 8 stations spread along 19.58 KM with varying travel times between the stations. The average rate of vehicle arrivals during peak hours is 15 minutes, and with an average velocity of 55 KM/H. According to the train capacity, the estimated average arrival rate of passengers for each station is approximately \( d_{i,j} = \frac{790 \text{ passengers}}{15 \text{ minutes}} = 56 \frac{\text{passengers}}{\text{hour}} \forall i \neq j \in S \).

5.1.3. Berlin Metro Trams – M2 & M4
The third and fourth case studies are two Berlin metro-trams characterized by medium number of stations and short distances between stations. Both metro-trams are using the Bombardier “Flexity” capable of carrying 240 passengers in the length of 40 meters. 200
million passengers use the 22 tram lines in Berlin yearly and between 10,000 to 25,000 passengers use the M-2, M-4 lines respectively (Peters, 2010).

The M-2 line consist of 17 stations spread along for 6.84 KM, with an approximately 1-minute drive between station and with an average velocity of 18.5 KM/h. The M-4 line consist of 25 stations spread along for 11.26 KM, with an approximately 1-minute drive between station and with an average velocity of 18.5 KM/h. According to the numbers of passengers per day, the estimated average arrival rate of passengers for each station is approximately for M-2: \( d_{i,j} = 15 \forall i \neq j \in S \), and for M-4 will be: \( d_{i,j} = 7 \forall i \neq j \in S \).

5.1.4. Bucharest – M41

The fifth case study is the Bucharest light-rail line characterized by medium number of stations and small distances between stations. The specific light-rail tram is using a segregated infrastructure to ensure fast travel times. The light-rail type is Bucur LF capable of carrying 240 passengers in the length of 40 meters.

The M-41 line consists of 15 stations spread over 9.72 KM, with an approximately 1-minute travel time between consecutive stations and with an average velocity of 19.5 KM/H. The average demand for M-41 is estimated as: \( d_{i,j} = 20 \forall i \neq j \in S \), representing 4200 passengers hour during peak hours.

5.1.5. Boston Blue & Orange Subway lines

The sixth and seventh case studies are the Boston subway characterized by medium number of stations and varied distances between stations. The blue line rolling stock is a Siemens 700-series, with a 15 meters car’s length, running on an infrastructure that consists of 12 stations spread along for 9.72 KM, with an approximately 2-minute drive between station and with an average velocity of 32.5 KMH. The average rate of vehicles arrivals is 4.5 minutes during peak hours.

The Orange line rolling stock is a CRRC subway trains, with a 20-meter car length, running on an infrastructure that consists of 20 stations distributed over 17.93 KM, with approximately 2-minute travel time between consecutive stations and with an average velocity of 30 KM/h. For the two lines, official data for the passenger demand matrix between stations was used, provided by Massachusetts Bay Transportation Authority (MBTA, 2023).
5.2. Results
For each case study we analyze the performance of the system for a range of fleet sizes. We present three main KPI’s: the mean wait to board, the percentage of aborting passengers and the mean trip occupancy. All of the results below describe feasible systems that ensure free flow of vehicles according to the Greenshields model. In particular, we have truncated the fleet ranges such that in none of the case studies and the considered fleet sizes, the system experienced over saturation. That is, the percentage of time in which the number of vehicles on a single segment (or more) exceeded the values prescribed by the Greenshielded model did no exceed 1%.

5.2.1. Metronit – Red Line
Figure 12 displays the mean wait time for a range of ACE-PRT fleet sizes (between 500 and 900). The wait times for the simulation model, the simulation unlimited capacity case, and the approximate model are represented in blue, gray and orange, respectively. In addition, the average peak service wait time in the existing service is represented by the green horizontal line. As can be expected, as the fleet size increases the average wait time decreases. Notably, for the tested fleet sizes, the proposed ACE-PRT service cannot obtain mean wait times that are lower than in the existing service. This confirms the insight from Section 3, that is, the station-to-station direct service is not likely to outperform fixed service over lines that consist of many stations.

Figure 13 displays the number of passengers served and the number of passengers who abort during a day. As can be observed, no passengers have aborted the system. That is, all passengers were served within an hour, but exhibited longer wait times on average, as compared to the current service. Lastly, Figure 14 displays the trip occupancy distribution. In particular, each curve represents the number of trips performed with a certain number of passengers on board (ranging from 1 to 10). Naturally, adding more vehicles to the system increases the number of low occupancy trips. In particular, with 900 vehicles, most trips are performed with 2-3 passengers on board.
Figure 12 - Average Waiting Time for Metronet System

Figure 13 - Number of Passenger Aborts for Metronet System
5.2.2. Israeli Suburban Train – Hod-Hasharon <> Hashalom Tel-Aviv

Figures 15-17 display the mean wait time, the number of aborts and the trip occupancy distribution for Israeli Suburban Train case. By placing more than 90 vehicles the proposed service is capable of outperforming the existing service, in terms of the mean wait times. Furthermore, with more than 90 vehicles the capacitated case and the un-capacitated cases converge, meaning that the 10 passenger capacity is not binding. A significant finding is that the approximated model, with more than 90 ACE-PRTs, is forecasting with a 99%+ accuracy the results of the ACE-PRT simulation model. According to Figure 16, no passengers are aborting the system when there are more than 70 vehicles. Lastly, Figure 17 shows that when installing more than 150 vehicles, the average passenger occupancy per trip goes down 1-3 passengers per vehicle.
Figure 15 - Average Waiting Time for Israel Suburban Train System

Figure 16 - Number of Passengers Aborts for Israel Suburban Train System
5.2.3. Bucharest Light-Rail Train

Figures 18-20 present the three KPIs for the Bucharest light rail case. As can be observed in Figure 18, approximately 300 vehicles are needed in order to obtain shorter wait time than in the current fixed service. The vehicle capacity becomes non binding as when more than 200 vehicles are distributed in the system. Furthermore, when there are more than 200 ACE-PRTs, ACE-PRT capacity does not impact the results of the model. It can also be observed that as the number of vehicles increase, the approximate model and the simulation model converge. Demonstrating once again, that the approximate model is very accurate when a sufficient number of vehicles is in use. Figure 19 shows that in the tested settings no passengers abort. Figure 20 shows that with more than 200 vehicles, in most trips, the vehicles are occupied by 2-3 passengers per vehicle.
Figure 18 - Average Waiting Time for Bucharest Light-Rail System

Average Waiting Time to ACE-PRT Vehicles
Bucharest Light-Rail Train - 15 stations, demand 20 pass/hour

Figure 19 - Number of Passengers Aborts for Bucharest Light-Rail System

Aborts - Bucharest Light-Rail Train
5.2.4. Berlin MetroTram M2/M4

Figures 21-23 and Figures 24-26 display the KPI’s for the Berlin Metro Tram M2 and the Berlin Metro Tram M4, respectively. Once again, with sufficient number of vehicles (200 for M2, 600 for M4) the proposed ACE-PRT service can obtain shorter wait times as compared to the existing services during peak hours. Although the number of stations of the M2 & M4 are the same, and the demand rate of passengers in M2 is higher, the performance of the ACE-PRT system is worse in the M4 system by the number of ACE-PRT needed to reach the same/better performance than the existing system. According to the data, this can be as a result of the total distance of the system which is twice as long as the M2 system. Furthermore, in both systems, ACE-PRT capacity does not impact the results of the model. At 200 vehicles in the M2 system, and at 600 ACE-PRT in the M4 system, the approximated model is forecasting with a 99%+ accuracy the results of the ACE-PRT simulation model. Lastly, Observing the Number of occupancy distributions (Figure 26, Figure 29), when adding more than 200 vehicles, the average number of passengers per trip goes down to mostly 1 passenger per vehicle.
Figure 21 - Average Waiting Time for Berlin MetroTram M2 System

Figure 22 - Number of Passengers Aborts for Berlin MetroTram M2 System
Figure 23 - Number of Passengers Per a Trip for Berlin MetroTram M2 System

Figure 24 - Average Waiting Time for Berlin MetroTram M4 System
5.2.5. **Boston Subway Blue/Orange Lines**

In the Boston Blue/Orange lines, the simulation & the approximated model ran on real-life demand data. The simulation model in the Blue line performed better than the existing service (Figure 27) whereas the simulation model in the Orange line had failed to perform better (Figure 30) & failed to serve all passengers (Figure 31). In both the Blue & Orange systems, the approximated model provided 99%+ accuracy to the simulation model where ACE-PRT capacity was 100. Unlike other systems that were examined in this document, when adding more ACE-PRT to the system, the number of ACE-PRT that host 4 passengers is increasing (Figure 29,32).
Figure 27 - Average Waiting Time for Boston Blue Line System

Figure 28 - Number of Passengers Aborts for Boston Blue Line System
Figure 29 - Number of Passengers Per a Trip Boston Blue Line System

Figure 30 - Average Waiting Time for Boston Orange Line System
5.2.6. Results Discussion:

In the previous section, the queueing theory based approximation model and the event-based simulation were compared. The two models converge when the number of vehicles to stations ratio is high enough and the ratio between the total offered capacity (number of vehicles times vehicle capacity) and the expected demand is high enough. When the latter ration is too low, an increase in the number of vehicles or in the vehicle capacity will shorten waiting time.
The lower the ratio between the number of vehicles and number of stations, it becomes more important to apply an intelligent empty vehicle repositioning policy. To conclude, the queueing model approximation predicts very well the performance of the system with uncapacitated vehicles.

To further enhance the approximate model, lower and upper bounds that are based on the capacity constraints and segment saturation constraints can be developed. Albeit, for finer results a simulation should be preferable. This bounds will be the feasible range of fleet sizes to operate without over-loading and over-saturating.

The event-based simulation proved that the ACE-PRT model could provide better waiting times than the current services’ peak hours waiting time when the number of stations is not relatively high. The best improvement in waiting time is shown in the suburban train with an improvement of 75% in waiting time.

The ACE-PRT model failed to outperform the current service in the Metronit, MetroTram line M4, and the Boston subway orange line cases, due to high number of stations leading to a high number of ACE-PRT vehicles and over-saturation. A possible good ridesharing policy might utilize the system and lower the number of ACE-PRT vehicles needed to satisfy the demand on those systems. When properly configured, the ACE-PRT approach was shown to be successful for all other case studies.

6. Conclusion and Further Research:
In this work, we examined the potential of transforming guideway based public transit to novel on-demand point-to-point services. The results show, that even with a simple vehicle routing policy that was analyzed in this research, it is possible to improve the level of service of passengers, particularly shortening waiting and journey time of passengers in certain use cases.

Three main assumptions in the model leave further room for significant improvements if properly optimized. The first assumption is that all vehicles are serving immediately a passenger that is waiting in a station without considering the option of waiting to other passengers to arrive in the station. Therefore, it might be that passengers, with the same origin station and destination station, that enters their origin station in a short time window, but separately, won’t be served by the same vehicle. The second assumption is the direct station to station policy which fails in systems with large number of stations. In that case, it is possible to consider clustering neighboring stations together, which in turn
may reduce the complexity and the number of low occupancy trips. The last assumption is the use of a simple reallocation policy, that can and should be optimized according to real-time data and more precise prediction according to the former behavior of passengers.

Lastly, the mathematical model is utterly important to quickly analyze the feasibility of the model, and deciding the proper number of vehicles that are needed to satisfy the demand. Having said that, the Israeli Queue Model is lacking the server capacity constraint and should be added as a further research.

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The advancement of technology in motion is altering the landscape of urban mobility. The automated vehicle, both within transport networks and in electric vehicles, is expected to revolutionize urban mobility through flexible shared services. This advancement will result in the sharing of vehicles and trips, allowing for better traffic control and increased accessibility for residents of systems that already exist. In this work, we examine the potential of converting existing public transportation systems (light rail, regional trains, electric and -BRT) into transportation service systems. To achieve this, we propose the use of small, automated vehicles that can be used on existing expensive networks that are not being utilized. We identify and characterize existing public transportation systems that could benefit from such a change. We develop tools to support decision-making for tactical and operational planning. In particular, we develop a model of the proposed service based on the "Israel's Tor" model. Such a representation enables fast and simple comparisons and takes into account the complexity of the system. Secondly, we suggest a more accurate representation of the system using an event-based simulation model. This simulation model enables a thorough examination of the various aspects of the system's operation and provides a deeper understanding of specific system settings. The results of this research show that it is possible to provide the current demand within a short waiting time for passengers in over 50% of the public transportation systems that were examined. Furthermore, using the "Israel's Tor" model, we are able to measure the waiting time of passengers in shared transportation services more accurately, compared to other mathematical models that are available in the literature.
ת推薦 המנהיגים

הנובר והנוברות גמר להוראה הסופית "מוסמך אוניברסיטה"
בנוכחות הנושאים
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המendir גמר במחוזה להוראה הסופית
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ת鲀ור האוטונומיה ע"פ דרישת עלי גבר ושחרת לקרמית

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